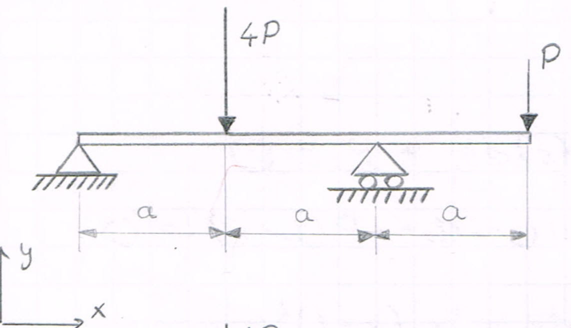
WYTRZYMAŁOŚĆ KONSTRUKCJI I

Seria VII

Maciej Jasiński

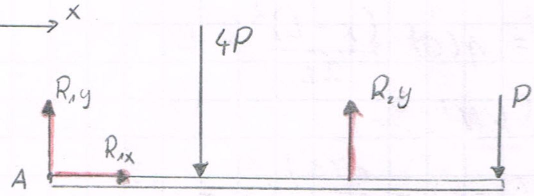
$I = 6 \quad N = 8$

Zadanie 1A



$P = 10{,}16\ kN$

$a = 1{,}16\ m$



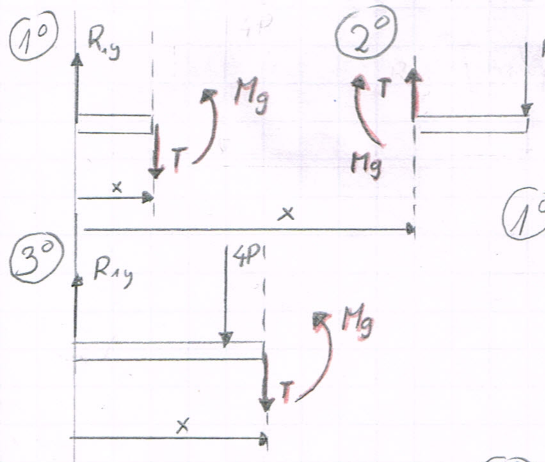
$\sum F_x = 0 \Rightarrow R_{1x} = 0$

$\sum F_y = 0 \Rightarrow 5P = R_{1y} + R_{2y}$

$\sum M_A = 0 \Rightarrow -4P\cancel{a} - 3P\cancel{a} + 2R_{2y}\cancel{a} = 0$

$2R_{2y} = 7P$

$R_{2y} = 3{,}5P \qquad R_{1y} = 1{,}5P$



1° $x \in (0, a)$

$T = R_{1y} = 1{,}5P$

$M_g = R_{1y} \cdot x = 1{,}5Px$

3° $x \in (a, 2a)$

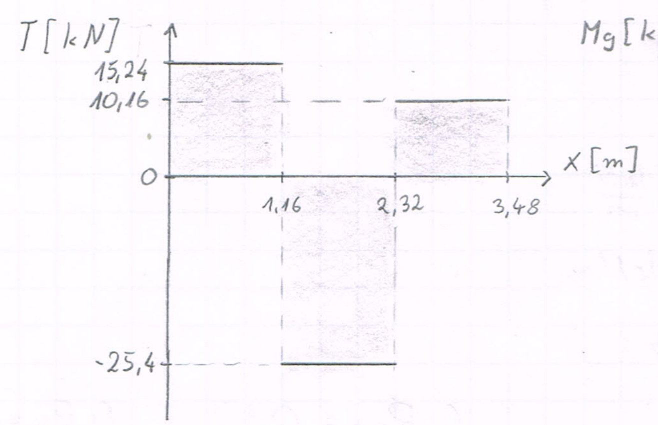
$T = R_{1y} - 4P = -2{,}5P$

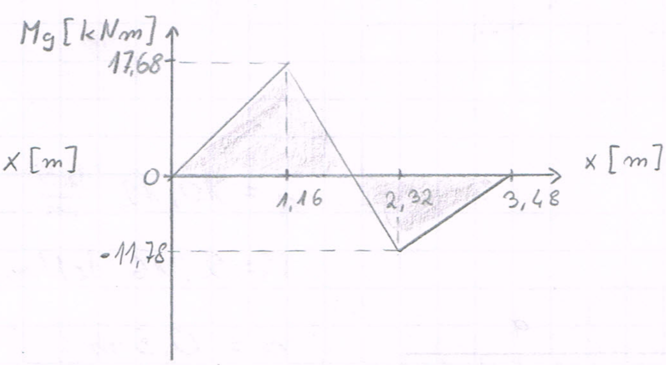
$M_g = R_{1y} \cdot x - 4P(x - a) = -2{,}5Px + 4$

2° $x \in (2a, 3a)$

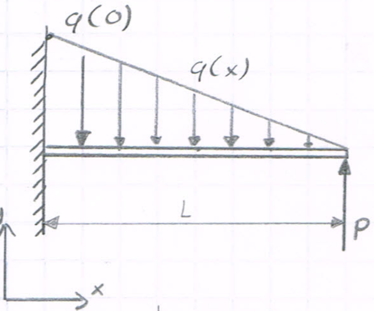
$T = P$

$M_g = -P(3a - x)$





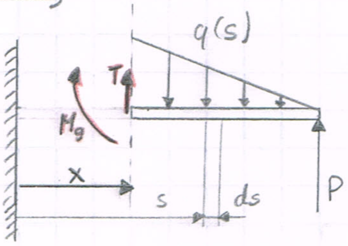
Zadanie 1B



$$q(0) = 5{,}16 \ \frac{kN}{m} \quad \Rightarrow \quad q(x) = 5{,}16 - 2{,}58\,x \quad \left[\frac{kN}{m}\right]$$

$$P = 4{,}12 \ kN \qquad q(x) = q(0) - \frac{q(0)}{L}\cdot x$$

$$L = 2m$$



$$\sum F_y = 0 \quad \Rightarrow \quad T - \int_x^L q(s)ds + P = 0$$

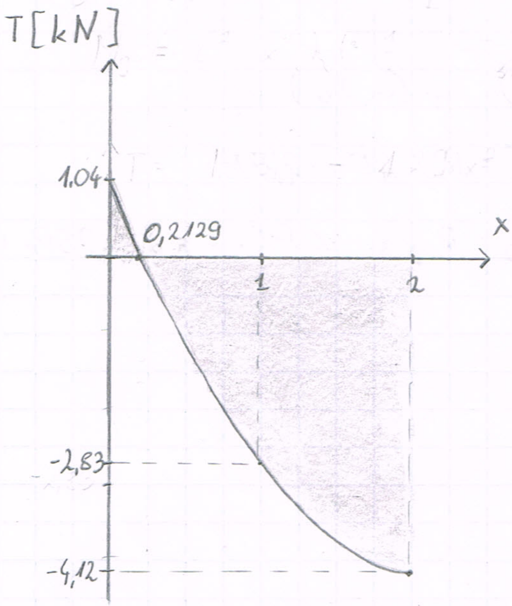
$$\sum M_x = 0 \quad \Rightarrow \quad -M_g - \int_x^L q(s)(s-x)ds + P(L-x) = 0$$

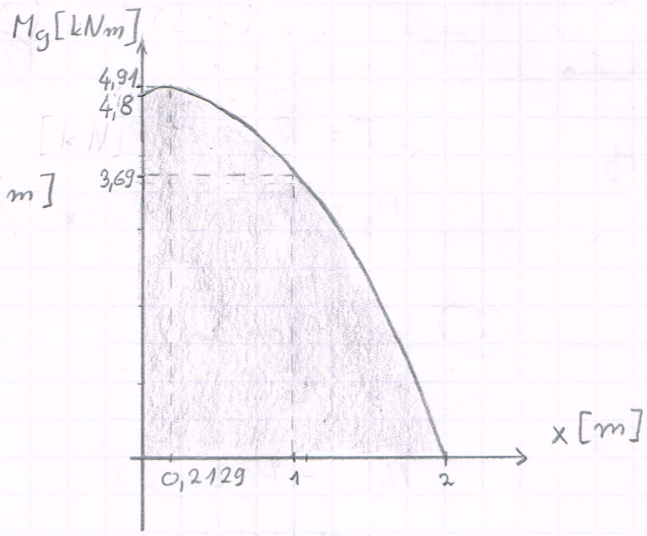
$$T = q(0)\int_x^L \left(1 - \frac{1}{L}s\right)ds - P = q(0)\,\frac{(x-L)^2}{2L} - P$$

$$T = 1{,}29(x-2)^2 - 4{,}12 \ [kN]$$

$$M_g = P(L-x) - q(0)\int_x^L \left(s - \frac{1}{L}s^2 - x + \frac{1}{L}xs\right)ds = P(L-x) - \frac{q(0)}{6L}(L-x)^3$$

$$M_g = 4{,}12(2-x) - 0{,}43(2-x)^3 \ [kNm]$$



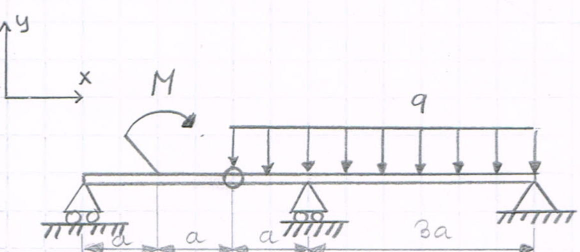


Zadanie 1C

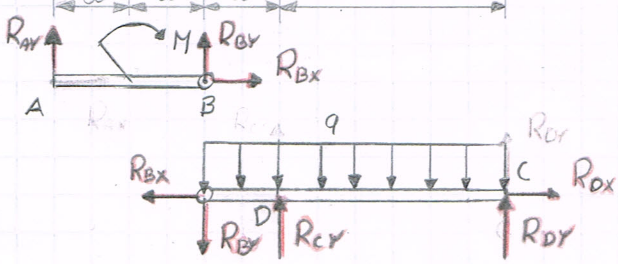
$$q = 10{,}12 \ \frac{kN}{m}$$

$$M = 2{,}16 \ kNm$$

$$a = 0{,}5m$$



$$\sum F_x = 0 \quad \Rightarrow \quad \begin{cases} R_{Bx} = 0 \\ R_{Dx} - R_{Bx} = 0 \end{cases} \quad \Rightarrow \quad \begin{cases} R_{Bx} = 0 \\ R_{Dx} = 0 \end{cases}$$



$$\sum F_y = 0 \quad \Rightarrow \quad \begin{cases} R_{AY} + R_{BY} = 0 \\ -R_{BY} + R_{CY} + R_{DY} - \int_{2a}^{6a} q\,dx = 0 \end{cases}$$

$$\sum M_B = 0 \Rightarrow \begin{cases} -M - R_{AY}\cdot 2a = 0 \\ R_{CY}\cdot a + R_{DY}\cdot 4a - \int_{2a}^{6a} q(x-2a)\,dx = 0 \end{cases}$$

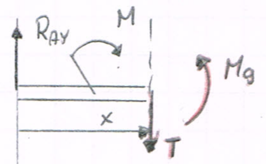
$$R_{AY} = -\frac{M}{2a} \qquad a R_{CY} + 4a R_{DY} - \left[\tfrac{1}{2}qx^2 - 2aqx\right]_{2a}^{6a} = 0$$

$$R_{BY} = \frac{M}{2a} \qquad \cancel{a} R_{CY} + 4\cancel{a} R_{DY} - 8qa^{\cancel{2}} = 0$$

$$R_{DY} = \frac{4}{3}aq - \frac{M}{6a} \qquad -R_{BY} + R_{CY} + R_{DY} - 4aq = 0$$

$$R_{CY} = \frac{8}{3}aq + \frac{2}{3}\frac{M}{a}$$

1°

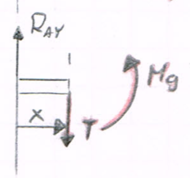


$x \in (a, 2a)$

$$T = R_{AY} = -\frac{M}{2a}$$

$$M_g = M + R_{AY}\cdot x = M - \frac{M}{2a}x$$

2°

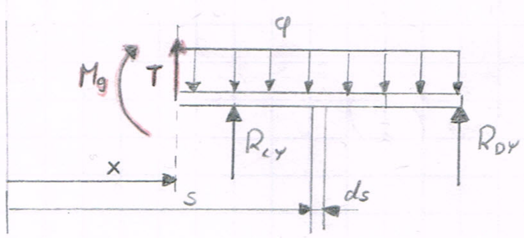


$x \in (0, a)$

$$T = R_{AY} = -\frac{M}{2a}$$

$$M_g = -\frac{M}{2a}x$$

3°



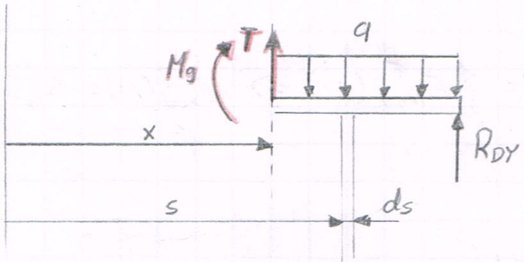
$x \in (2a, 3a)$

$$T = -R_{CY} - R_{DY} + \int_x^{6a} q\,ds = 2aq - \frac{M}{2a} - xq$$

$$M_g = (3a-x)R_{CY} + (6a-x)R_{DY} - \int_x^{6a} q(s-x)\,ds$$

$$M_g = -2a^2q + M + 2aqx - \frac{M}{2a}x - \frac{1}{2}qx^2$$

4°

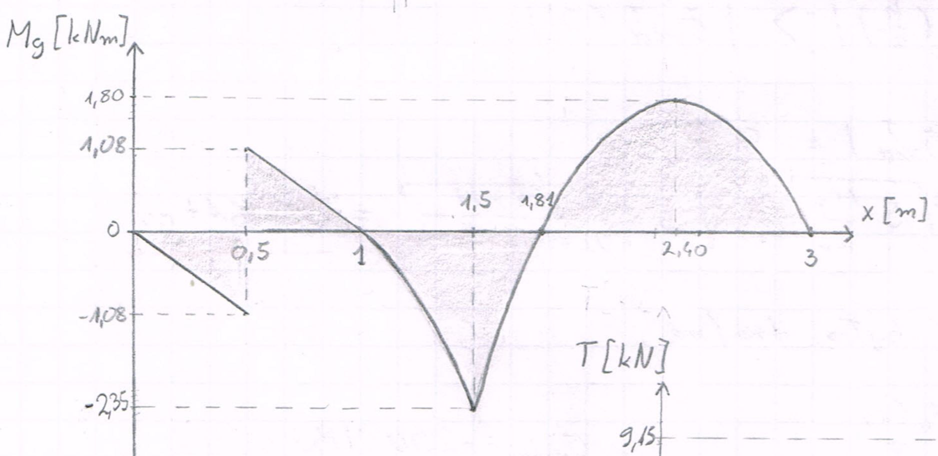


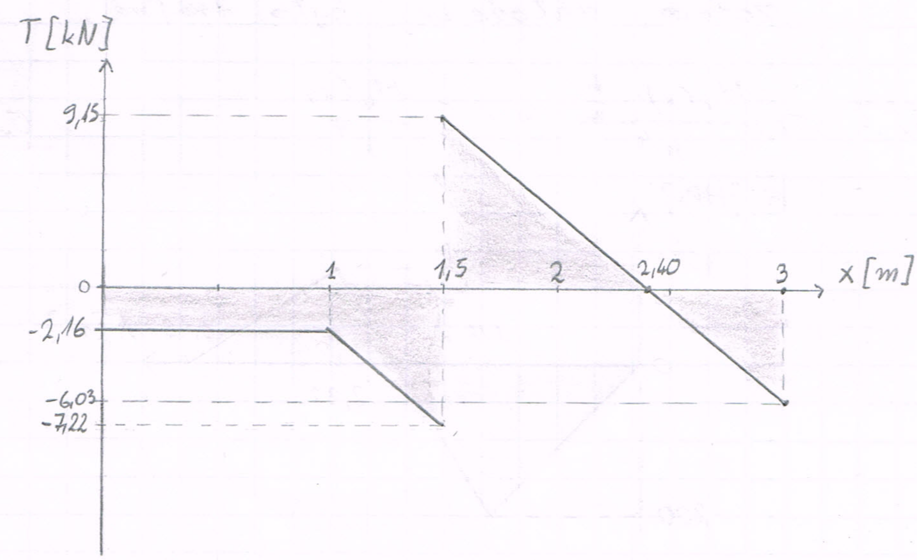
$x \in (3a, 6a)$

$$T = -R_{DY} + \int_x^{6a} q\,ds = \frac{14}{3}aq + \frac{M}{6a} - xq$$

$$M_g = (6a-x)R_{DY} - \int_x^{6a} q(s-x)\,ds$$

$$M_g = -10a^2q - M + \frac{14}{3}aqx + \frac{M}{6a}x - \frac{1}{2}qx^2$$





$R_{AY} = -2,16$ kN

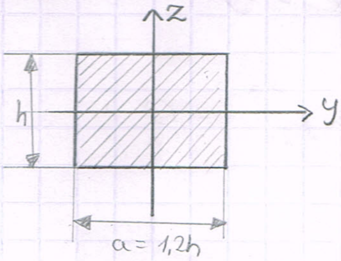
$R_{BY} = 2,16$ kN

$R_{CY} = 16,37$ kN

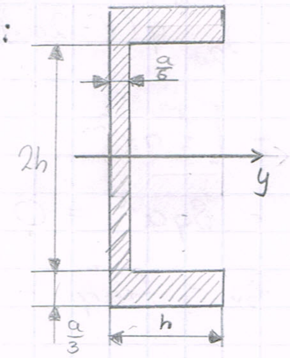
$R_{DY} = 6,03$ kN

Zadanie 2.

I:



II:



$k_r = 200\,MPa$

→ Najbardziej wytężony przekrój występuje dla $x \to 1{,}16^{+}\,m$

wtedy: $T = -25{,}4\,kN$

$M_g = 17{,}68\,kNm$

I:

$$J_y = \int_A z^2\,dA = \frac{1}{12}h^3 a = \frac{1}{10}h^4$$

$$S_y(z) = \int_A z\,dA = \int_{-\frac{a}{2}}^{\frac{a}{2}}\int_z^{\frac{h}{2}} z\,dz\,dy = \frac{1}{2}a\left(\frac{h^2}{4} - z^2\right)$$

$$\sigma(z) = -\frac{M_g z}{J_y} = -10\frac{M_g z}{h^4}$$

$$\tau(z) = \frac{T S_y(z)}{J_y \cdot a} = \frac{T\cdot\frac{1}{2}\cancel{a}\left(\frac{h^2}{4} - z^2\right)}{\frac{1}{10}h^4\cdot\cancel{a}} = 5\frac{T\left(\frac{h^2}{4} - z^2\right)}{h^4}$$

$$\begin{cases}\sigma\left(\frac{h}{2}\right) = -\cancel{10}^{\,5}\dfrac{M_g\cdot\frac{\cancel{h}}{2}}{h^{\cancel{4}3}} = -5\dfrac{M_g}{h^3} = -88{,}4\dfrac{1}{h^3} = \sigma_{red}^T\left(\frac{h}{2}\right)\\ \tau\left(\frac{h}{2}\right) = 0\end{cases}$$

$$\begin{cases}\sigma(0) = 0\\ \tau(0) = 5T\dfrac{h^{\cancel{2}}}{4}\cdot\dfrac{1}{h^{\cancel{4}2}} = \dfrac{5}{4}\dfrac{T}{h^2} = -31{,}75\dfrac{1}{h^2} = \dfrac{1}{2}\sigma_{red}^T(0) \Rightarrow \sigma_{red}^T(0) = -63{,}5\dfrac{1}{h^2}\end{cases}$$

Załóżmy, że $h < 1$

$$\left|\sigma_{red}^T\left(\frac{h}{2}\right)\right| > \left|\sigma_{red}^T(0)\right|$$

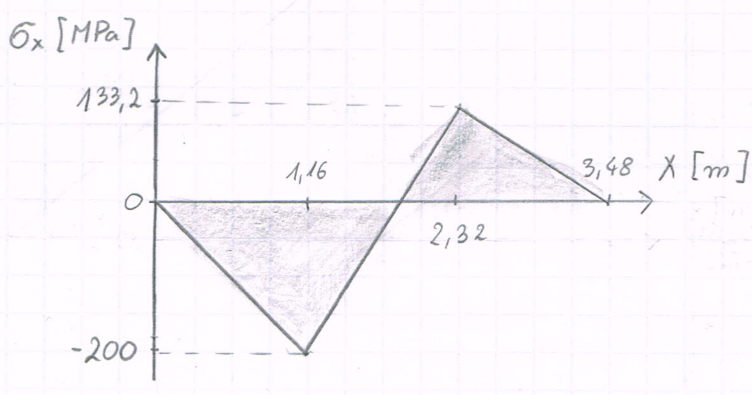
$$k_r \geq \left|\sigma_{red}^T\right| = 5\frac{M_g}{h^3}$$

$$h \geq \sqrt[3]{5\frac{M_g}{k_r}} = \sqrt[3]{5\cdot\frac{17{,}68\,kN}{200\,000\,kPa}} = 0{,}07617\,m$$

zatem założenie było trafne.

$$\sigma_x^{extr} = -10\frac{M_g(x)\cdot\frac{h}{2}}{h^4} = -5\frac{M_g(x)}{h^3}$$

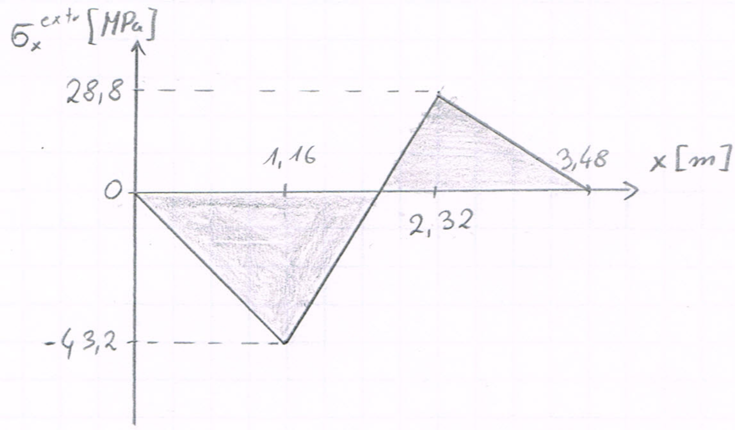
$\sigma_{max} = -200\,MPa$



II:

$$J_y = \frac{1}{12} h \left(2h + \frac{2}{3}a\right)^3 - \frac{1}{12}\left(h - \frac{a}{6}\right)\cdot(2h)^3 = \frac{1}{12} h (2{,}8h)^3 - \frac{1}{12}\cdot 0{,}8h(2h)^3 =$$
$$= 1{,}296\, h^4$$

$$\sigma_x^{extr} = -\frac{M_g(x)\, z^{extr}}{J_y} = -\frac{M_y(x)\left(h + \frac{a}{3}\right)}{J_y} = -\frac{M_g(x)\cdot 1{,}4h}{1{,}296h^4} \approx -1{,}080\,\frac{M_g(x)}{h^3}$$



$\sigma_{max} = -43{,}2$ MPa

$$\boxed{\frac{\sigma_{max\,I}}{\sigma_{max\,II}} = 4{,}63}$$

Zatem drugi przykład budowy belki jest 4,63 razy wytrzymalszy na zginanie, co czyni go znacznie lepszym wykorzystaniem masy.